Exercise 11

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$x^3 + \sqrt{x} = \int_0^x \frac{1}{(x-t)^{\frac{5}{6}}} u(t)\, dt$$

Solution

This is a Volterra integral equation because one of the limits of integration is not constant. It is of the first kind because the unknown function u appears only inside the integral. It's inhomogeneous because of the $x^3 + \sqrt{x}$. It is singular since the integrand becomes infinite at a point $t = x$ in the interval of integration. This equation in particular is known as a generalized Abel integral equation since the exponent of $x - t$ in the denominator is $5/6$, not $1/2$.